TRINITY COLLEGE

Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section Two: Calculator-assumed **65% (99 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

A seafood processor buys batches of n prawns from their supplier, where n is a constant. In any given batch, the probability that a prawn is export quality is p, where p is a constant and the quality of an individual prawn is independent of other prawns.

The discrete random variable X is the number of export quality prawns in a batch and the mean of X is 79.2 and standard deviation of X is 6.6.

(a) State the name given to the distribution of X and determine its parameters n and p. (4 marks)

(b) Determine the probability that more than 50% of prawns in a randomly selected batch are export quality. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 10 **(8 marks)**

The voltage, V volts, supplied by a battery t hours after timing began is given by

$$V = 8.95e^{-0.265t}$$

(a) Determine

(i) the initial voltage. (1 mark)

(ii) the voltage after 3 hours. (1 mark)

(iii) the time taken for the voltage to reach 0.03 volts. (1 mark)

(b) Show that $\frac{dV}{dt} = aV$ and state the value of the constant a. (2 marks)

(c) Determine the rate of change of voltage 3 hours after timing began. (1 mark)

(d) Determine the time at which the voltage is decreasing at 5% of its initial rate of decrease. (2 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 11 **(8 marks)**

A small body moving in a straight line has displacement x cm from the origin at time t seconds given by

$$x = 5\cos(2t - 1) + 6.5, \qquad 0 \leq t \leq 3.$$

(a) Use derivatives to justify that the maximum displacement of the body occurs when $t = 0.5$. (4 marks)

(b) Determine the time(s) when the velocity of the body is not changing. (2 marks)

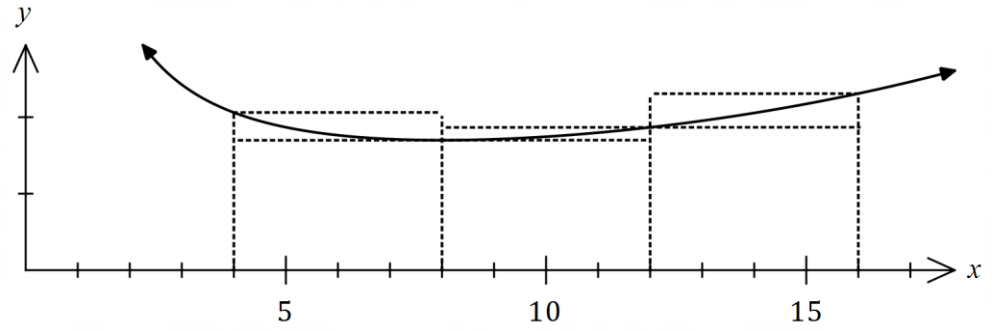
(c) Express the acceleration of the body in terms of its displacement x. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 12 **(7 marks)**

The function f is defined as $f(x) = \frac{5e^{0.125x}}{x}$, $x > 0$, and the graph of $y = f(x)$ is shown below.



(a) Complete the missing values in the table below, rounding to 2 decimal places. (1 mark)

x	4	8	12	16
$f(x)$		1.70	1.87	

(b) Use the areas of the rectangles shown on the graph to determine an under- and over-estimate for $\int_4^{16} f(x)\,dx$. (3 marks)

(c) Use your answers to part (b) to obtain an estimate for $\int_4^{16} f(x)\,dx$. (1 mark)

(d) State whether your estimate in part (c) is too large or too small and suggest a modification to the numerical method employed to obtain a more accurate estimate. (2 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 13 **(8 marks)**

A bag contains four similar balls, one coloured red and three coloured green. A game consists of selecting two balls at random, one after the other and with the first replaced before the second is drawn. The random variable X is the number of red balls selected in one game.

(a) Complete the probability distribution for X below. (3 marks)

x	0	1	2
$P(X = x)$			

(b) Determine $E(X)$ and $\text{Var}(X)$. (2 marks)

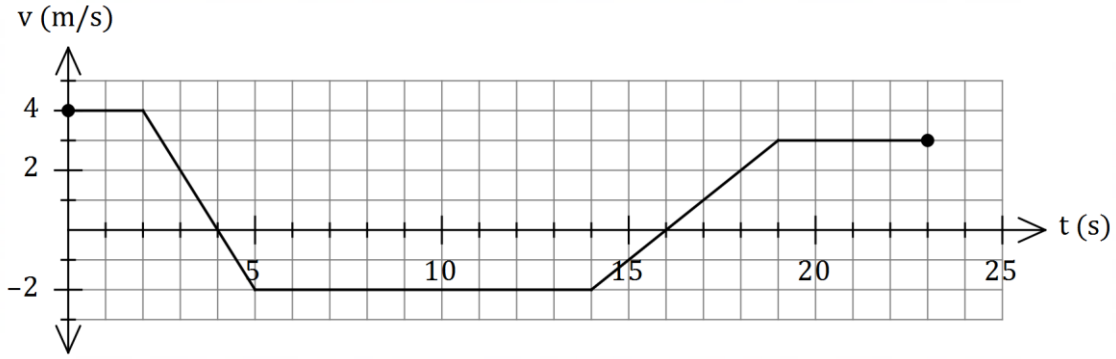
(c) A player wins a game if the two balls selected have the same colour. Determine the probability that a player wins no more than three times when they play five games. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 14 **(9 marks)**

A small body leaves point A and travels in a straight line for 23 seconds until it reaches point B. The velocity v m/s of the body is shown in the graph below for $0 \leq t \leq 23$ seconds.



(a) Use the graph to evaluate $\int_0^4 v\,dt$ and interpret your answer with reference to the motion of the small body. (3 marks)

(b) Determine an expression, in terms of t, for the displacement of the body relative to A during the interval $2 \leq t \leq 5$. (3 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(c) Determine the time(s) at which the body was at point A for $0 < t \leq 23$. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 15 **(7 marks)**

Functions f and g are such that

$$f(2) = -1, \qquad f'(x) = 6(2x-7)^{-2}$$

$$g(-3) = -1, \qquad g'(x) = 6(2x+7)^{-2}$$

(a) Determine $f(3)$. (3 marks)

(b) Use the increments formula to determine an approximation for $g(-2.97)$. (3 marks)

(c) Briefly discuss whether using the information given about f and the increments formula would yield a reasonable approximation for $f(3)$. (1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 16 **(9 marks)**

When a machine is serviced, between 1 and 5 of its parts are replaced. Records indicate that 7% of machines need 1 part replaced, 8% need 5 parts replaced, 12% need 4 parts replaced, and the mean number of parts replaced per service is 2.82.

Let the random variable X be the number of parts that need replacing when a randomly selected machine is serviced.

(a) Complete the probability distribution table for X below. (4 marks)

x	1	2	3	4	5
$P(X = x)$					

(b) Determine $\text{Var}(X)$. (2 marks)

The cost of servicing a machine is $\$56$ plus $\$12.50$ per part replaced and the random variable Y is the cost of servicing a randomly selected machine.

(c) Determine the mean and standard deviation of Y. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 17 **(6 marks)**

Some values of the polynomial function f are shown in the table below:

x	-2	-1	0	1	2	3	4
$f(x)$	-8	0	5	6	4	1	-3

(a) Evaluate $\int_1^4 f'(x)\,dx$. (2 marks)

The following is also known about $f'(x)$:

Interval	$-2 \leq x \leq 1$	$x = 1$	$1 \leq x \leq 4$
$f'(x)$	$f'(x) > 0$	$f'(x) = 0$	$f'(x) < 0$

(b) Determine the area between the curve $y = f'(x)$ and the x-axis, bounded by $x = -2$ and $x = 3$. (4 marks)

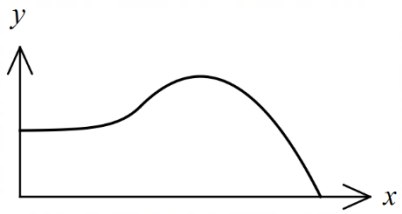
DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 18 **(6 marks)**

The edges of a swimming pool design, when viewed from above, are the x-axis, the y-axis and the curves

$y = -0.2x^2 + 3x - 6.25$ and $y = 2.75 + e^{x-5}$

where x and y are measured in metres.



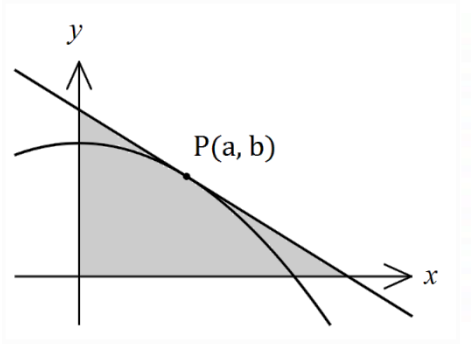
(a) Determine the gradient of the curve at the point where the two curves meet. (2 marks)

(b) Determine the surface area of the swimming pool. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 19 **(8 marks)**

Let $P(a, b)$ be a point in the first quadrant that lies on the curve $y = 8 - x^2$ and A be the area of the triangle formed by the tangent to the curve at P and the coordinate axes.



(a) Show that the equation of the tangent formed at P is given by $y = -2ax + a^2 + 8$. (2 marks)

(b) Hence or otherwise show that $A = \dfrac{(a^2 + 8)^2}{4a}$. (2 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(c) Use calculus to determine the coordinates of P that minimise A. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Question 20 **(9 marks)**

A farmer introduced a species of trout into his dam. The number of trout, P, t years after they were introduced is modelled by the equation $P = 80e^{0.2t}$ where $t \geq 0$.

(a) How many trout were initially introduced to the dam? (1 mark)

(b) Determine the number of years taken for the population of trout to first exceed 1000. (2 marks)

(c) Calculate $\frac{dP}{dt}$ when $t = 5$ and explain the value you have calculated with reference to the context of the question.

(3 marks)

(d) The farmer also introduced yabbies into his dam at the same time as the trout. The farmer introduced 200 yabbies and it is known the yabby population had a rate of change given by $\frac{dP}{dt} = 0.07t$. After how many years would the farmer expect the population of the trout and yabbies to be equal.

(3 marks)

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 21 **(8 marks)**

When a byte of data is sent through a network in binary form (a sequence of bits - 0's and 1's), there is a chance of bit errors that corrupt the byte, i.e. a 0 becomes a 1 and vice versa.

Suppose a byte consists of a sequence of 8 bits and for a particular network, the chance of a bit error is 0.300%.

(a) Determine the probability that a byte is transmitted without corruption, rounding your answer to 5 decimal places. (3 marks)

(b) Determine the probability that during the transmission of 32 bytes, at least one of the bytes becomes corrupted. (2 marks)

A Hamming code converts a byte of 8 bits into a byte of 12 bits for transmission, with the advantage that if just one bit error occurs during transmission, it can be detected and corrected.

(c) Determine the probability that during the transmission of 32 bytes using Hamming codes, at least one of the bytes becomes permanently corrupted. (3 marks)

End of questions

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF